作业7

7.1

3 What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?

9 What is the probability that a five–card poker hand does not contain the queen of hearts?

17 What is the probability that a five–card poker hand contains a straight, that is, five cards that have consecutive kinds? (Note that an ace can be considered either the lowest card of an A–2–3–4–5 straight or the highest card of a 10–J–Q–K–A straight.)

28 In a superlottery, a player selects 7 numbers out of the first 80 positive integers. What is the probability that a person wins the grand prize by picking 7 numbers that are among the 11 numbers selected at random by a computer.

38 A player in the Mega Millions lottery picks five different integers between 1 and 70 , inclusive, and a sixth integer between 1 and 25, inclusive, which may duplicate one of the earlier five integers. The player wins the jackpot if all six numbers match the numbers drawn.

a) What is the probability that a player wins the jackpot?

b) What is the probability that a player wins $ 1,000,000, the prize for matching the first five numbers, but not the sixth number, drawn?

c) What is the probability that a player wins $ 500, the prize for matching exactly four of the first five numbers, but not the sixth number, drawn?

d) What is the probability that a player wins $10, the prize for matching exactly three of the first five num–bers but not the sixth number drawn, or for match–ing exactly two of the first five numbers and the sixth number drawn?

42Two events E_1 and E_2 are called independent if $p\left(E_1 \cap E_2\right)=p\left(E_1\right) p\left(E_2\right)$. For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.
a) E_1 : tails comes up with the coin is tossed the first time; E_2 : heads comes up when the coin is tossed the second time.
b) E_1 : the first coin comes up tails; E_2 : two, and not three, heads come up in a row.c)

c) E_1 : the second coin comes up tails; E_2 : two, and not three, heads come up in a row.
(We will study independence of events in more depth in Section 7.2.)

7.2

1 What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?

5 A pair of dice is loaded. The probability that a 4 appears on the first die is 2/7, and the probability that a 3 appears on the second die is 2/7. Other outcomes for each die appear with probability 1∕7. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled?

10 What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

a) The first 13 letters of the permutation are in alphabetical order.

b) a is the first letter of the permutation and z is the last letter.

c) a and z are next to each other in the permutation.

d) a and b are not next to each other in the permutation.

e) a and z are separated by at least 23 letters in the permutation.

f) z precedes both a and b in the permutation.

12 Suppose that E and F are events such that $p(E) = 0.8$ and $p(F) = 0.6$. Show that $p(E \cup F) \geq 0.8$ and . $p(E \cap F) \geq 0.4$

19 a) What is the probability that two people chosen at random were born during the same month of the year?

b) What is the probability that in a group of n people chosen at random, there are at least two born in the same month of the year?

c) How many people chosen at random are needed to make the probability greater than 1 / 2 that there are at least two people born in the same month of the year?

28 Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has

a) exactly three boys?

b) at least one boy?

c) at least one girl?

d) all children of the same sex?

35 Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

a) the probability of no failures

b) the probability of at least one failure

c) the probability of at most one failure

d) the probability of at least two failures

38 A pair of dice is rolled in a remote location and when you ask an honest observer whether at least one die came up six, this honest observer answers in the affirmative.

a) What is the probability that the sum of the numbers that came up on the two dice is seven, given the information provided by the honest observer?

b) Suppose that the honest observer tells us that at least one die came up five. What is the probability the sum of the numbers that came up on the dice is seven, given this information?

7.3

1 Suppose that E and F are events in a sample space and $p(E)=1/3, p(F)=1/2$, and $p(E \mid F)=2/5$. Find $p(F \mid E)$

8 Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.9% of people with the disease test positive and only 0.02% who do not have the disease test positive.

a) What is the probability that someone who tests posi–tive has the genetic disease?

b) What is the probability that someone who tests nega–tive does not have the disease?

9 Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that

a) a patient testing positive for HIV with this test is infected with it?

b) a patient testing positive for HIV with this test is not infected with it?

c) a patient testing negative for HIV with this test is infected with it?

d) a patient testing negative for HIV with this test is not infected with it?

10 Suppose that 4% of the patients tested in a clinic are infected with avian influenza. Furthermore, suppose that when a blood test for avian influenza is given, 97% of the patients infected with avian influenza test positive and that 2% of the patients not infected with avian influenza test positive. What is the probability that

a) a patient testing positive for avian influenza with this test is infected with it?

b) a patient testing positive for avian influenza with this test is not infected with it?

c) a patient testing negative for avian influenza with this test is infected with it?

d) a patient testing negative for avian influenza with this test is not infected with it?

15 In this exercise we will use Bayes' theorem to solve the Monty Hall puzzle (Example 10 in Section 7.1). Recall that in this puzzle you are asked to select one of three doors to open. There is a large prize behind one of the three doors and the other two doors are losers. After you select a door, Monty Hall opens one of the two doors you did not select that he knows is a losing door, selecting at random if both are losing doors. Monty asks you whether you would like to switch doors. Suppose that the three doors in the puzzle are labeled 1,2 , and 3 . Let W be the random variable whose value is the number of the winning door; assume that $p(W=k)=1/3$ for $k=1,2,3$. Let M denote the random variable whose value is the number of the door that Monty opens. Suppose you choose door i.

a) What is the probability that you will win the prize if the game ends without Monty asking you whether you want to change doors?

b) Find $p(M=j \mid W=k)$ for j=1,2,3 and k=1,2,3

c) Use Bayes' theorem to find $p(W=j \mid M=k)$ where i and j and k are distinct values.

d) Explain why the answer to part (c) tells you whether you should change doors when Monty gives you the chance to do so.

7.4

2 What is the expected number of heads that come up when a fair coin is flipped 10 times?

7 The final exam of a discrete mathematics course con–sists of 50 true/false questions, each worth two points, and 25 multiple–choice questions, each worth four points. The

probability that Linda answers a true/false question correctly is 0.9, and the probability that she answers a

multiple–choice question correctly is 0.8. What is her ex–pected score on the final?

16 Let X and Y be the random variables that count the num–ber of heads and the number of tails that come up when two fair coins are flipped. Show that X and Y are not in–dependent.

19 Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped.

a) What is the expected value of X_n ?

b) What is the variance of X_n ?

Test 1

1. What is the probability that a fair coin lands heads four times out of five flips?
2. What is the probability that a positive integer less than 100 picked at random has all distinct digits?
3. Suppose that two cards are drawn without replacement from a well–shuffled deck. What is the probability that both cards have numbers and that the numbers on the cards are the same (note that only the numbers 2 through 10 are shown on cards, since aces, kings, queens, and jacks are represented by letters).
4. A fair red die and a fair blue die are rolled. What is the expected value of the sum of the number on the red die plus three times the number on the blue die?
5. Two identical urns contain balls. One of the urns has 6 red balls and 3 blue balls. The other urn has 5 red balls and 8 blue balls. An urn is chosen at random and a ball is drawn at random from this urn. If the ball turns out to be red, what is the probability that this is the urn with 6 red balls?

Test

1. A computer picks out at random a sequence of six digits.
   (a) What is the probability that a person picks all six digits in their correct positions?
   (b) What is the probability that a person picks exactly five of the digits, in the correct order?
2. What is the probability that in a group of 200 random people, at least two of them have the same triple of initials (such as RSZ for Ruth Suzanne Zeitman), assuming that

each triple of initials is equally likely. Give the answer as a calculation; it is not necessary to evaluate the expression.

3. Suppose that a bag contains six slips of paper: one with the number 1 written on it, two with the number 2,and three with the number 3. What is the expected value and variance of the number drawn if one slip is selected at random from the bag?
4. What is the probability that a random person who tests positive for a certain blood disease actually has the disease, if we know that 1% of the population has the disease, that 95% of those who have the disease test positive for it, and 2% of those who do not have the disease test positive for it.
5. Two identical urns contain balls. One of the urns has 6 red balls and 3 blue balls. The other urn has 5 red balls and 8 blue balls. An urn is chosen at random and two balls are drawn at random from this urn,without replacement.
   (a) What is the probability that both balls are red?
   (b) What is the probability that the second ball is red, given that the first ball is red?